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A Nonvarying-C^* Control Scheme for Aircraft

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Introduction

THE usual method for accommodating an automatic flight control system to the wide variations in dynamic characteristics of the airframe, with changes of airspeed and altitude over its flight envelope, is to change control-loop gains with measurements of air data. The dynamic pressure, or the Mach number, as estimated by an air data computer, is generally used as the parameter with which the gains are scheduled. Our system provides a response that is uniform for all flight conditions without using measurements of dynamic pressure, altitude or angle of attack. The scheme is very close to that used for SIDAC.[1,2] The modification is based on the observation that the equation for handling qualities C^* criterion is very similar to a basic short-period equation for the motion of the aircraft. A modest feedback and feedforward with variable gains holds the coefficients of this equation fixed. The C^* requirement may be met by choosing these coefficients to be the same as demanded by the Criterion or by adding a fixed outer loop. The mechanism for varying the gains is found by a gradient calculation similar to that used in the SIDAC analysis. We achieve the advantage of a low-gain, narrow-bandwidth system which is very insensitive to instrument nose and bending modes, accommodates the primary control system, and satisfies the requirements directly rather than using a model-following technique.

The SIDAC system identifies parameters and uses this in turn to adjust gains. Since the accuracy of identifying the several coefficients depends on the frequency content of the motion and the particular flight condition, it appears that a system which calls for the required response directly should have an advantage. This has also been argued by Hofmann and Best[3] in a fairly similar approach to control of the lateral-directional axes.

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C^*-Criterion

The most difficult problem in flight control design, besides considerations of making the system invulnerable to component failures, is in deciding on what flying characteristics will be acceptable to the pilot. The requirement for longitudinal response which best fits into an analytical formulation is that promulgated by Tobie, Elliot, and Malcom.[4] The criterion is that the time-response trace of a quantity called C^*, for an abrupt force applied to the stick, must fall within a certain envelope. C^* is the sum of the normal force applied to the pilot's seat plus a constant multiple of the angular velocity in pitch. Thus, it is a linear combination, with constant positive coefficients, of normal acceleration at the aircraft's center of gravity, the pitch velocity and the pitch acceleration, assuming the pilot's station is ahead of the center of gravity.

Development

We begin with the equations of perturbations from straight and level flight written as

$$\begin{aligned} \ddot\theta &= M_q\dot\theta + M_\alpha\alpha + M_{\dot\alpha}\dot\alpha + M_\delta\delta \\ n &= U_0(\dot\theta - \dot\alpha) = -Z_\alpha\alpha - Z_\delta\delta \end{aligned} \tag{1}$$

in terms of angular rate in pitch $\dot\theta$, angle of attack α, and normal acceleration at the center of gravity n. The quantity U_0 is the value of the aircraft's unperturbed velocity and δ represents the elevator deflection. The coefficients M_δ, Z_α, and Z_δ are constants, representative of the flight condition, and have appropriate dimensions.

Angle of attack is difficult to measure, and its interpretation is complicated by turbulence in the air, so, following Shipley,[1] we algebraically eliminate it from the equations and find

$$\begin{aligned} \ddot\theta &= \bar\beta_q\dot\theta + \bar\beta_n n + \bar\beta_\delta\delta \\ \dot n &= -Z_\alpha\dot\theta + (Z_\alpha/U_0)n - Z_\delta\dot\delta \end{aligned} \tag{2}$$

The first equation in $\equiv$ 2 is the fundamental relation in this study. The C^* quantity is

$$C^* = n + l\ddot\theta + U_c\dot\theta \tag{3}$$

in which l is the distance of the pilot forward of the center of gravity and U_c is a number called the cross-over velocity, usually taken around 400 fps. If we require C^* to be precisely a multiple of the command input C_c,

$$C^* = -kC_c \tag{4}$$

Equation (3) may be written as

$$\ddot\theta = -(U_c/l)\dot\theta - (1/l)n - (k/l)C_c \tag{5}$$

This is exactly the form of the fundamental equation in Eq. (2).

The control configuration is diagrammed in Fig. 1. The equations are

$$\begin{aligned} \ddot\theta &= \bar\beta_q\dot\theta + \bar\beta_n n + \bar\beta_\delta[(C+f)/(T_aS+1)] \\ \epsilon &= \ddot\theta - \beta_q\dot\theta - \beta_n n - \beta_\delta[C/(T_aS+1)] \\ C &= C_c + H_{\dot q}\ddot\theta + H_q\dot\theta + H_n n \end{aligned} \tag{6}$$

Since the LaPlace operator is equivalent to differentiation, the first two equations may be rewritten as

$$\begin{aligned} (T_aS+1)\epsilon &= (T_aS+1)\ddot\theta - T_a\beta_q\dot\theta - T_a\beta_n\dot n - \beta_n n - \beta_\delta C \\ (T_aS+1)\ddot\theta &= T_a\bar\beta_q\ddot\theta + \bar\beta_q\dot\theta + T_a\bar\beta_n\dot n + \bar\beta_n n + \bar\beta_\delta(C+f) \end{aligned} \tag{7}$$

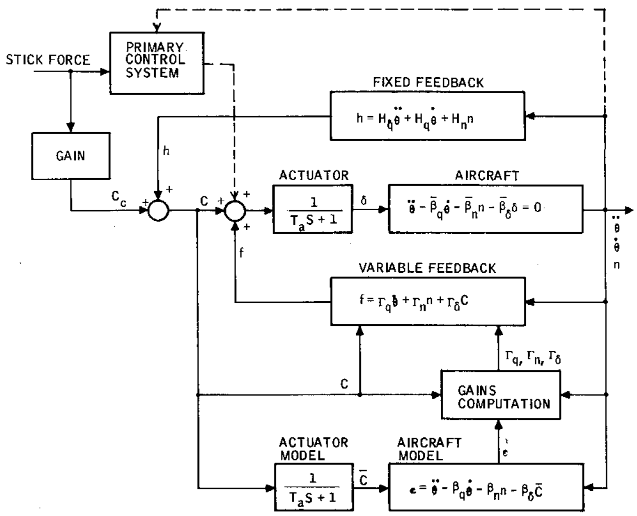

Fig. 1 Control configuration.

Combining these two and including the evaluation of the control function gives

$$(T_aS + 1)\epsilon = T_a(\bar{\beta}_q - \beta_q)\ddot{\theta} + T_a(\bar{\beta}_n - \beta_n)\dot{n} + (\bar{\beta}_q - \beta_q + \bar{\beta}_\delta\Gamma_q)\dot{\theta} + (\bar{\beta}_n - \beta_n + \bar{\beta}_\delta\Gamma_n)n + (\bar{\beta}_\delta - \beta_\delta + \bar{\beta}_\delta\Gamma_\delta)C \quad (8)$$

The actuator introduces the first two terms on the right-hand side which could be taken into account by adding two more feedbacks of $\ddot{\theta}$ and $\dot{n}$ with varying gains. We avoid this complication and hope that these terms will not lead to serious errors. Analog simulation shows that this is the case.

The requirement given by Eq. (4), that C^* should be a multiple of the command input is unrealistic, considering the vast latitude by the criterion. We modify this by requiring that C^* follow the command by a simple lag,

$$C^* = [-k/(T_cS + 1)]C_c \quad (9)$$

and calculate the deviation from this by

$$\epsilon_c = C^* + [-k/(T_cS + 1)]C_c \quad (10)$$

On expanding this and applying some algebra we arrive at the following choice of values for the fixed-gain feedbacks:

$$\begin{aligned} H_n &= -(1 + \mu\beta_n)/\mu\beta_\delta \\ H_q &= -(U_c + \mu\beta_q)/\mu\beta_\delta \\ H_q &= -(l + T_cU_c - \mu + \mu T_a\beta_q)/\mu\beta_\delta \end{aligned} \quad (11)$$

The parameter μ is an arbitrary number which is picked later by analog analysis.

The gain changers are chosen by a gradient technique,[5-7] to drive the error term ϵ to zero. They operate from the equations

$$\begin{aligned} d\Gamma_q/dt &= K_q\dot{\theta}G \\ d\Gamma_n/dt &= K_n nG \\ d\Gamma_\delta/dt &= K_\delta CG \end{aligned} \qquad G = \operatorname{sgn}\epsilon = \begin{cases} +1, \epsilon > \epsilon_0 \geqq 0 \\ 0, -\epsilon_0 \leqq \epsilon \leqq \epsilon_0 \\ -1, \epsilon < -\epsilon_0 \end{cases} \quad (12)$$

Performance of the System

The controller was evaluated on an analog simulation. After initial study with only the actuator as the complication, a representation of the primary control system, a second-order 40 rad/sec servo, a simulation of 0.1° elevator hysteresis, and equations describing a bending of 0.1° elevator hysteresis, and equations describing a bending mode were added. The system accepted these although the rates and accuracies for the convergence of the gains were reduced. The dynamics of the second-order servo introduce limit cycles under relatively large command inputs. The effects of noisy measurements and air turbulence were studied. The system was not upset by these. The most notable aspect of the performance was the insensitivity of the C^* response to large deviations of the varying gains from their ideal values.

The fixed outer feedback could be eliminated by choosing the parameters of the error expression β_q, β_n, β_δ to be equal to the corresponding values in the C^* Equation. The simulation showed that with this choice the gain-changing mechanism was unstable. To keep the loop gains and the errors introduced by neglected complications small, these parameters are picked to be in the range of the corresponding aircraft parameters $\bar{\beta}_q$, $\bar{\beta}_n$, $\bar{\beta}_\delta$. Then the system is stable.

Conclusion

The Nonvarying-C^* Control scheme has been shown to have practical application for fixed-wing aircraft. It should be possible to extend the method of use on helicopters. The approach of choosing the variable-gain mechanism on the basis of system performance rather than requiring an explicit parameter results in a low-bandwidth system that is very tolerant to noise and high-order effects.

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